When $\nu = 0$, the relation $L_1 = L_2$ holds and the control $u = -x_1 - 0.01\, x_1$ will ensure the asymptotic stability of the system. When $\nu = 1$, we have $L_1 \subset L_2$ and the control will render the system unstable. This can be confirmed by considering the corresponding characteristic polynomial. Therefore the law of control ensuring the asymptotic stability of a mechanical system in a vacuum will not necessarily guarantee it in a viscous medium.

REFERENCES

1. SOKOLOV B. N., Bounds on the control in the linear dynamic optimization problem with a quadratic functional. *Prikl. Mat. Mekh.* **54**, 4, 1990.
2. SOKOLOV B. N., The minimum dimensions of the control vector in the linear dynamic problem of stabilization. *Prikl. Mat. Mekh.* **54**, 5, 1990.
3. SOKOLOV B. N., Stabilization of dynamic systems with geometrically constrained control. *Prikl. Mat. Mekh.* **55**, 1, 1991.
4. KRASOVSKII N. N., *Theory of the Control of Motion*. Nauka, Moscow, 1968.
5. BRYSON A. E. Jr. and YU-CHI HO, *Applied Optimal Control, Optimization, Estimation and Control.* Hemisphere, Washington, DC, 1975.

Translated by L.K.

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THE EQUILIBRIUM OF A PARABOLIC–LOGARITHMIC SHELL OF REVOLUTION†

G. I. NAZAROV and A. A. PUCHKOV

Kiev

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An exact general analytic solution is constructed for static membrane equations of equilibrium, in a complex form, for a parabolic–logarithmic shell of revolution with variable external load.

1. BASIC FORMULAS

STATIC momentless (mean brake) equilibrium of the middle surface of an elastic shell of revolution is described, in geographic coordinates z, θ, by the following system of partial differential equations:

$$\frac{\partial}{\partial z}(rT_1) - r'T_2 + t\frac{\partial S}{\partial \theta} + rtX_1 = 0 \tag{1.1}$$

$$t\frac{\partial T_2}{\partial \theta} + r\frac{\partial S}{\partial z} + 2r'S + rtX_2 = 0$$

$$t^2T_2 - rr''T_1 + rt^3Z = 0 \quad (t = \sqrt{1 + r'^2})$$

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Here T_1, T_2 are the internal forces directed, respectively, along the tangents to the parallel and meridian of the middle surface of the shell of revolution, S is the shear force, X_1, X_2, Z are the components of external forces, and $r = r(z)$ is a given meridian of the shell.

The formulas connecting the forces T_1, T_2, S with the stress functions $\varphi(z, \theta)$ and $\psi(z, \theta)$ [1] can be written for the case $X_1 = X_2 = 0$ $Z = N(z)$ in the form

$$T_1 = t\left(\frac{\psi}{r} + t^2 \frac{N}{2r''}\right), \quad T_2 = \frac{r''\psi}{t} - \frac{rtN}{2}, \quad S = \frac{\varphi}{r^2} \tag{1.2}$$

Substituting them into (1.1) we obtain the following equations:

$$\frac{d}{dz}\left(\frac{rt^3N}{r''}\right) + rr'tN = 0 \tag{1.3}$$

$$\frac{\partial\varphi}{\partial z} - P(z)\frac{\partial\psi}{\partial\theta} = 0, \quad \frac{\partial\varphi}{\partial\theta} + Q(z)\frac{\partial\psi}{\partial z} = 0 \tag{1.4}$$

$$P = -rr'', \quad Q = r^2 \tag{1.5}$$

Equations (1.4) are identical with those discussed in [2] for the case $X_1 = X_2 = Z = 0$, and are equivalent to the following two second-order equations:

$$\begin{gathered} r^2 \frac{\partial^2\psi}{\partial z^2} - r''\frac{\partial^2\psi}{\partial\theta^2} + 2r'\frac{\partial\psi}{\partial z} = 0 \\ rr''\frac{\partial^2\varphi}{\partial z^2} - (r'')^2\frac{\partial^2\varphi}{\partial\theta^2} - (r r'' + rr''')\frac{\partial\varphi}{\partial z} = 0 \end{gathered} \tag{1.6}$$

Depending on the sign of the Gaussian curvature, system (1.4) and Eqs (1.6) are either elliptical ($r''<0$, $P>0$) or hyperbolic ($r''>0$, $P<0$).

The above equations must be supplemented by the corresponding tangential boundary conditions at the shell edges.

The ordinary differential equation for the function $N(z)$ (1.3) has now been separated from the first of (1.1), and the third equation of (1.1) is identically satisfied.

Integrating (1.3) we obtain the following expression:

$$N = Dr''/(rt^4) \tag{1.7}$$

which holds for any sign of the Gaussian curvature of the middle surface. Here D is the constant of integration. The function (1.7) determines the external normal pressure ($D>0$) or the internal normal pressure ($D<0$) relative to the shell.

We note that when $D = -1$, function (1.7) is identical with the expression for the Gaussian curvature.

Let us now consider a parabolic–logarithmic shell of revolution of positive Gaussian curvature ($r''<0$), with the meridian

$$r = af^{1/2}\ln f \quad (f = cz + b) \tag{1.8}$$

We place the origin of coordinates at its apex, and direct the axis of symmetry vertically downwards. Let $0 \leqslant z \leqslant H$ where H is the height of the shell.

An indeterminacy of the type $0\cdot\infty$ which appears is removed at the point $z_0(cz_0 + b = 0)$ and this yields the relation $r(z_0) = 0$. A throat of radius $r_0 = ab^{1/2}\ln b$ appears at the apex $z = 0$, and the shell (1.8) has two edges while its curvature is nearly zero.

2. GENERAL SOLUTION

We will seek the solution of system (1.4) with the shell (1.8) in the form [3]

$$\varphi = \varphi_0 + \mathrm{Re}\,[\alpha(z)\,W(\zeta) + \textstyle\int W(\zeta)\,d\zeta], \quad \psi = \psi_0 + \mathrm{Im}\,\beta(z)\,W(\zeta) \tag{2.1}$$

$$\zeta = \int \sqrt{P/Q}\, dz + i\theta \tag{2.2}$$

Here φ_0, ψ_0 are constants and $W(\zeta)$ is an arbitrary analytic function of complex argument.

The real functions $\alpha(z)$ and $\beta(z)$ of a single argument z must be sought such that system (1.4) is satisfied for any function $W(\zeta)$. To do this, we introduce into it the correspondence derivatives of expressions (2.1).

We take into account the properties of analytic functions of a complex argument

$$\operatorname{Re}\,[if(\zeta)] = -\operatorname{Im} f(\zeta), \quad \operatorname{Im}\,[if(\zeta)] = \operatorname{Re} f(\zeta)$$

and omit the signs accompanying the real and imaginary parts:

$$(\sqrt{Q}\alpha' + \sqrt{P})\, W + \sqrt{P}\,(\alpha - \sqrt{PQ}\beta)\, W' = 0$$
$$(1 - Q\beta')\, W + (\alpha - \sqrt{PQ}\beta)\, W' = 0$$

The above relations are satisfied for any function $W(\zeta)$ only in the case when two functions $\alpha(z)$ and $\beta(z)$ simultaneously satisfy the following three equations:

$$\alpha = -\int \sqrt{P/Q}\, dz, \quad \beta = \int Q^{-1}\, dz, \quad \alpha = \sqrt{PQ}\,\beta \tag{2.3}$$

The constants of integration are unimportant and are therefore omitted.

Let us substitute the functions (1.5) for the shell (1.8) into Eqs (2.3), and integrate the result. This yields

$$\alpha = -{}^1\!/_2 \ln f, \quad \beta = -\varkappa \quad \varkappa = 1/(ca^2 \ln f) \tag{2.4}$$

and the third equation of (2.3) holds.

Formulas (2.1), taking relations (2.4) into account, now take the form

$$\varphi = \varphi_0 + \operatorname{Re}\left[-{}^1\!/_2 \ln fW(\zeta) + \int W\, d\zeta\right], \quad \psi = \psi_0 - \varkappa \operatorname{Im} W(\zeta) \tag{2.5}$$

and the complex variable (2.2) will become

$$\zeta = {}^1\!/_2 \ln f + i\theta$$

Formulas (2.5) for the shell (1.8) represent the general solution of system (1.4) and Eqs (1.6) and this can be confirmed by direct substitution.

If we take as the function $W(\zeta)$ the convergent exponential series [3] (here and henceforth the summation in n will be carried out from $n = 1$ to ∞):

$$W = \sum (A_n + iB_n)\, e^{n\omega\zeta} + (C_n + iD_n) e^{-n\omega\zeta}$$

where $\omega = 1$ for a closed shell ($-\pi \leqslant \theta \leqslant \pi$) and $\omega = \pi/\lambda$ for an open shell ($0 \leqslant \theta \leqslant \lambda$, $\lambda = \text{const}$); A_n, B_n, C_n, D_n ($n = 1, 2, \ldots$) are arbitrary constants, then after substituting it into (2.5) and separating the real and imaginary parts, we obtain for the case $\omega = 1$:

$$\varphi = \varphi_0 + \sum (A_n\alpha_n - C_n\beta_n) \cos n\theta - (B_n\alpha_n + D_n\beta_n) \sin n\theta,$$
$$\psi = \psi_0 - \varkappa \sum (B_n f^{n/2} + D_n f^{-n/2}) \cos \theta + (A_n f^{n/2} - C_n f^{-n/2}) \sin n\theta \tag{2.6}$$
$$\alpha_n = \frac{1}{2n}(2 - n \ln f)^{n/2}, \quad \beta_n = \frac{1}{2n}(2 + n \ln f)\, f^{-n/2}$$

Here the $4n$ constants appearing in (2.6) are found with the aid of the usual method of Fourier expansion in $\sin n\theta$ and $\cos n\theta$ in the interval $-\pi \leqslant \theta \leqslant \pi$, of the given tangential values of the angle θ of the function ψ and φ, or, which is the same, of the forces T_1, S at different edges of the shell.

The forces (1.2), (2.6) obtained in this manner will serve as initial forces for solving the essentially independent problem of displacements, whose geometrical equations are of the same type as system (1.4) [1].

In conclusion, we note that the functions (2.1) can be written in the form of a differential operator [3], provided that we put $W = F'(\zeta)$ where $F(\zeta)$ is an arbitrary analytic function of the argument ζ (2.2).

REFERENCES

1. GOL'DENVEIZER A. L., *Theory of Thin Elastic Shells*. Nauka, Moscow, 1976.
2. VLASOV V. Z., *The General Theory of Shells and its Applications in Technology*. Gostekhizdat, Moscow, 1949.
3. NAZAROV G. I., The exact solution of the equations of gas dynamics. *Izv. Akad. Nauk. SSSR, MZhG* 3, 1968.

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INTERACTION OF DISLOCATIONS IN AN ANISOTROPIC MEDIUM†

S. V. KUZNETSOV

Moscow

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THE INTERACTION of two dislocations at a distance from each other, in an anisotropic medium with elastic anisotropy of general type is considered and the forces acting on the dislocation defects determined. The solution is constructed using the method of multipolar expansions.

The mechanical properties of crystalline bodies depend to considerable degree on the presence of defects within them and their interaction with each other. The energy of interaction between the defects is the basic factor determining their mutual distribution and orientation within the crystal.

The publications available in this field deal either with the determination of the energy at which an isolated dislocation loop appears [1–3], with the associated forces acting on an isolated defect [4] and with the stress fields near the dislocation loop [5], or with the study of the interaction of dislocation defects between each other [6–10] or with foreign inclusions [11–13].

In spite of the fact that the majority of crystals are elastically anisotropic, the investigations leading to the determination of the energy of interaction between the dislocations were carried out, basically, for isotropic of transversally isotropic (hexagonal) crystals. This can be explained by the need to use the fundamental solutions of the equations of equilibrium, whose derivatives are used, in the majority of cases, to express the energy of interaction between the dislocations.

The fundamental solutions of the equations of equilibrium are constructed in closed form for isotropic media (Kelvin solution) and for a subclass of orthotropic materials, which includes transversally isotropic media, in

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